

An approach to the study of time, time-frequency and time-scale transformations for seismic migration problems

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Content

Antecedents

Research project ECOPETROL-COLCIENCIAS

Seismic pre-stack migration in depth by extrapolating wave fields using high performance computing for massive data in complex areas.

General objective

Develop and implement specialized algorithms for processing massive seismic data to obtain images with excellent focusing of energy, definition of structures and preservation of the attributes of the seismic data, to ensure more reliable interpretation processes in order to reduce exploration risk.

Antecedents

Research project ECOPETROL-COLCIENCIAS

Seismic pre-stack migration in depth by extrapolating wave fields using high performance computing for massive data in complex areas.

Specific objective

Develop algorithms of seismic migration using wave field extrapolation in the direction of time (RTM-Reverse Time Migration), evaluating the preservation of amplitudes and frequencies as well as the conditions of stability, numerical dispersion and computational cost.

Antecedents

Research project ITM-EAFIT

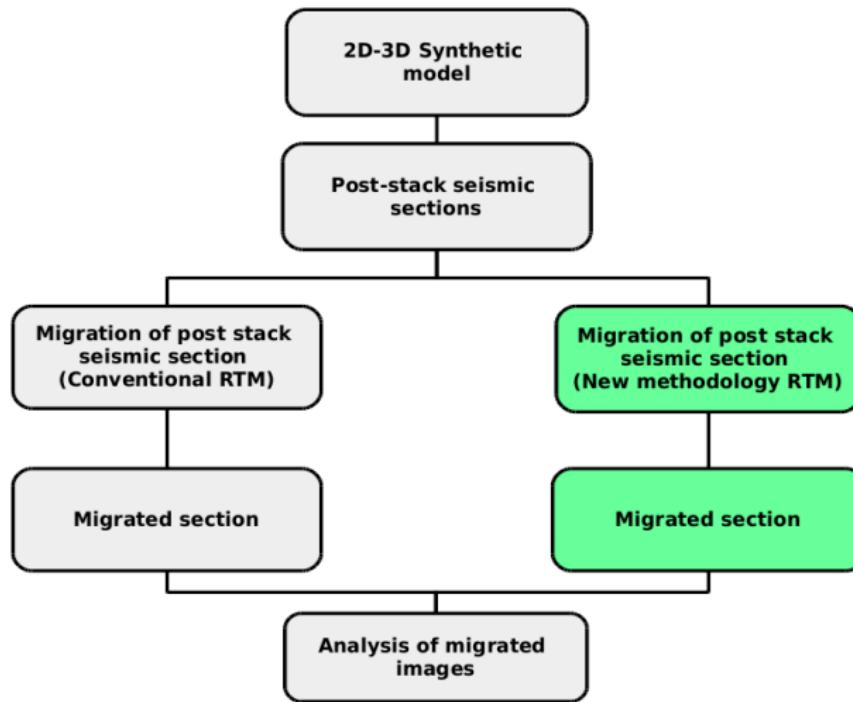
Study of Mathematical Modeling techniques and Functional Analysis in 2D-3D Reverse Time Migration. .

General objective

To develop and implement algorithms for the analysis and processing of massive seismic data of complex areas using the techniques of integral linear transformations and wavelet transformation system, looking for to approximate the system defined by the subsoil; in order to improve the results of imaging by seismic migration for use in oil exploration.

Methodology

First step



Methodology

Second step

Applying the new methodology for massive seismic data (2D, 3D) and computational improvement.

Third step

Applying the new methodology (First step) to pre-stack seismic data (2D, 3D).

Background

Wavelet analysis

Methods for the analysis in the frequency-time domain (scale). Analysis of non-stationary signals or strong changes in small intervals.

They are used in:

- Geophysics ([9], [28])
- Astrophysics ([56])
- Biology ([48])
- Signal and image in Medicine ([7],[51])
- Compression fingerprints and images ([6])

Background

Wavelet analysis

- Satellite images ([16])
- Atmospheric analysis and turbulence ([31],[62], [26])
- Processing pressure transient signals ([28])
- Among others

Background

A brief history

- Gabor transform, 1946. [30].
- Gabor transform modified with dilated Windows, 1969. [22]
- Morlet wavelet basis, 1982. [49]
- Goupillaud, 1984, [36].
- Orthogonal wavelet transform and pyramid algorithm, 1988 – 1989.[15], [45], [46]
- Seismic data compression and satellite transmission, 1995.[44],[5], [19], [57]
- Emergence of new orthogonal wavelet transforms, 2005. [8], [42]

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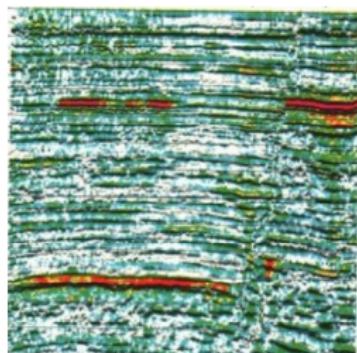
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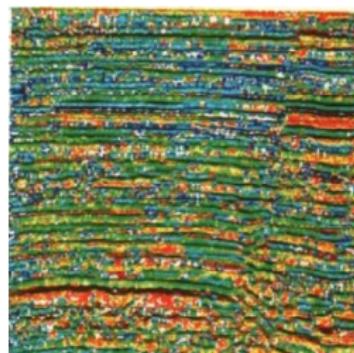
Background

Wavelets in Geology and Geophysics

- Complex seismic trace analysis, 1979. [61]



Reflection strength



Instantaneous frequency

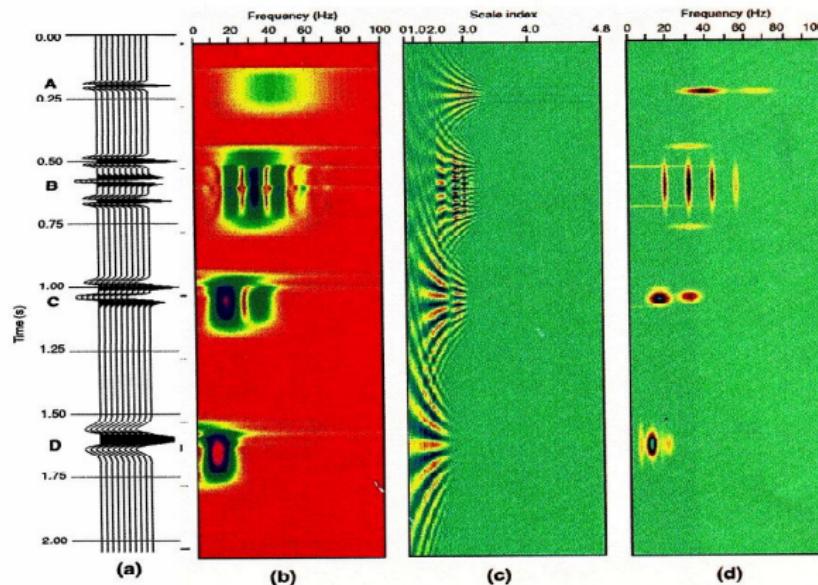


Apparent polarity

Background

Wavelets in Geology and Geophysics

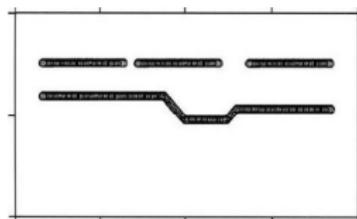
- STFT, CWT, MPD, 1995. [9]



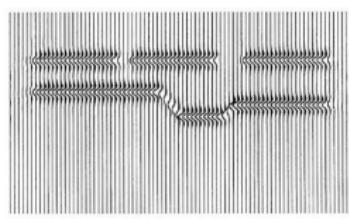
Background

Wavelets in Geology and Geophysics

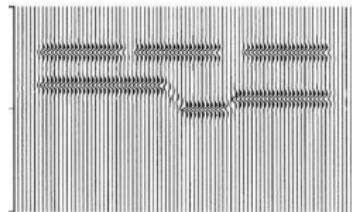
- Acoustic wavelet transform, 1998. [66]



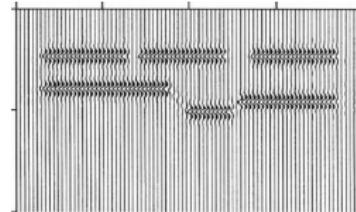
Synthetic model



Daubechies-4



Daubechies-8

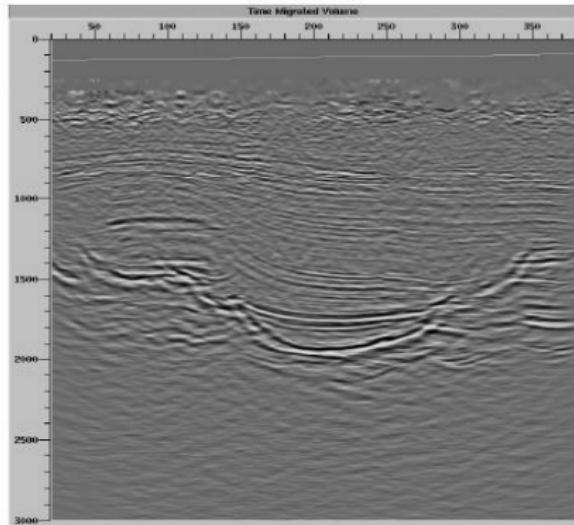


Daubechies-12

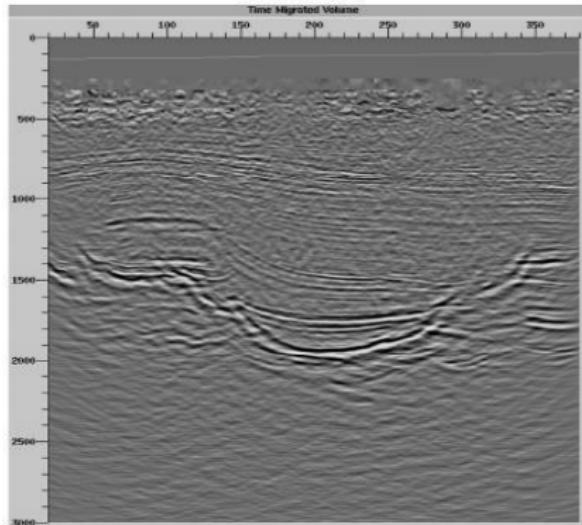
Background

Wavelets in Geology and Geophysics

- Fast Kirchhoff migration, 2002. [69]



Conventional prestack migration

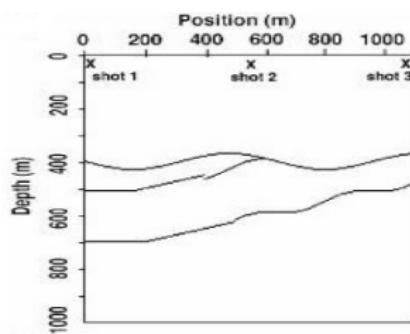


Wavelet prestack migration

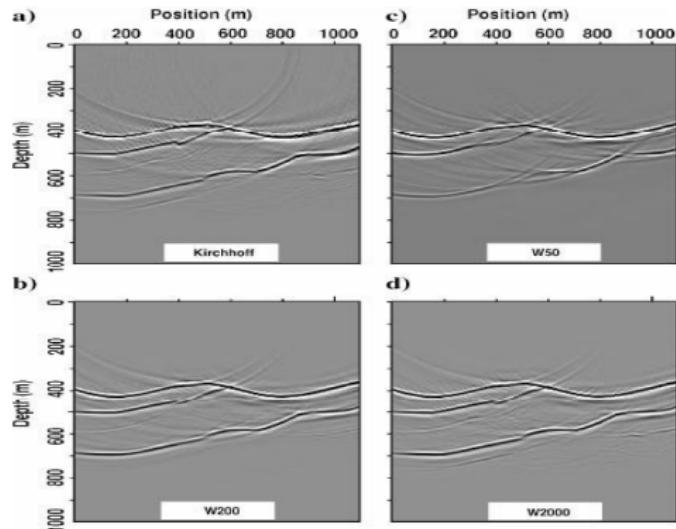
Background

Wavelets in Geology and Geophysics

- Prestack multiscale Kirchhoff migration, 2004.[67]



Reflectivity model

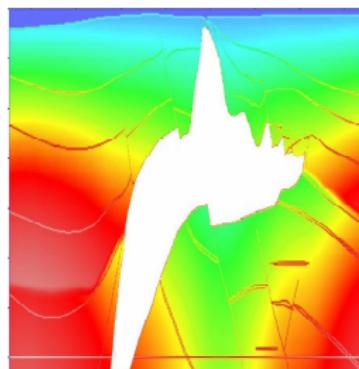


Migrated image

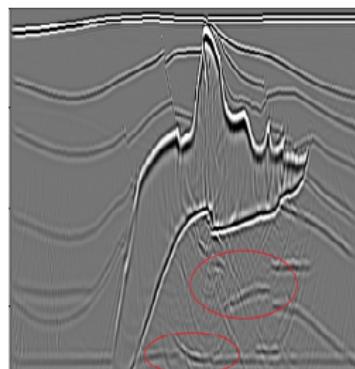
Background

Wavelets in Geology and Geophysics

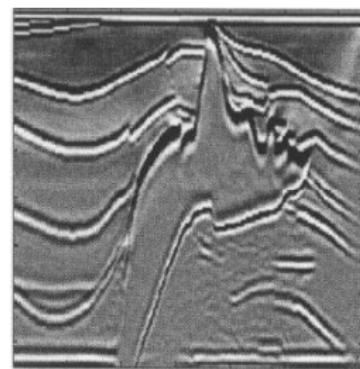
- Beamlet prestack depth migration, 2004. [12]



SEG-EAGE salt model



Split step migration

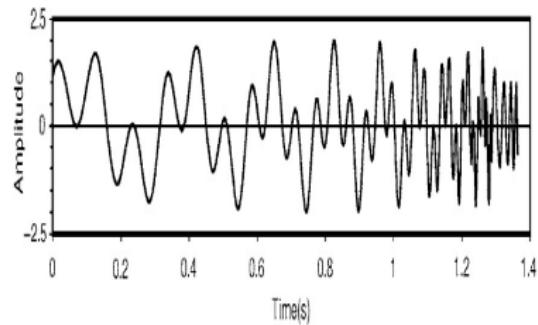


Beamlet prestack depth migration

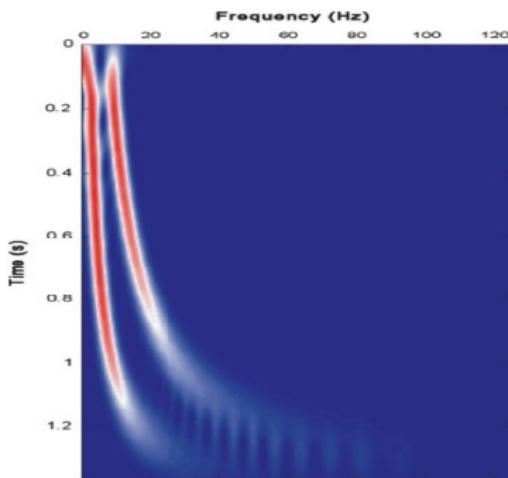
Background

Wavelets in Geology and Geophysics

- Time frequency continuous wavelet transform TFCWT, 2005.
[55]



Chirp signal

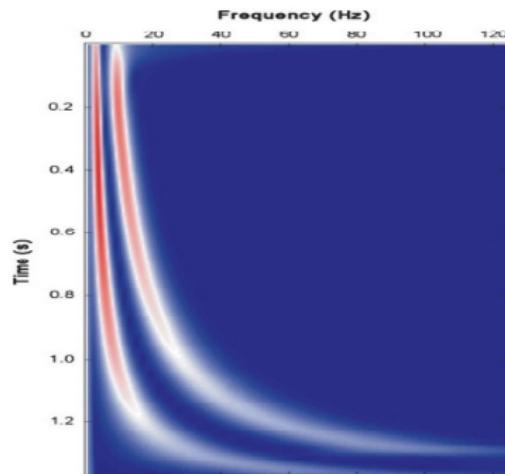


SFFT spectrum

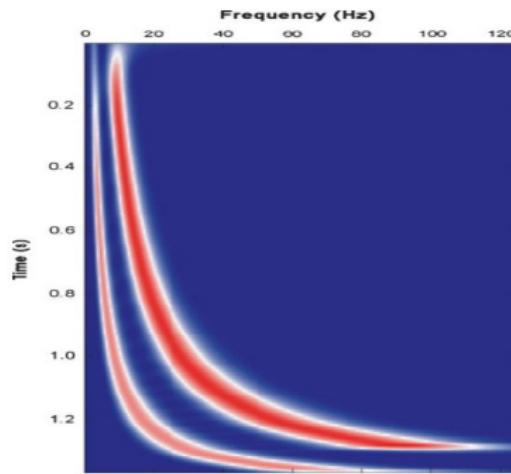
Background

Wavelets in Geology and Geophysics

- Time frequency continuous wavelet transform TFCWT. [55]



CWT spectrum

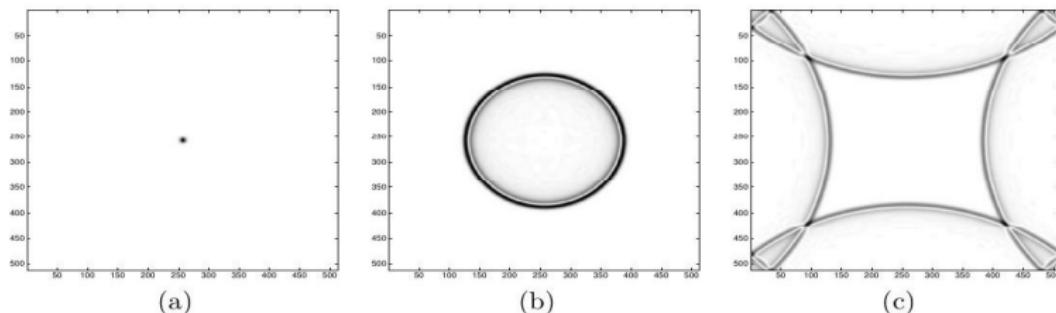


TFCWT spectrum

Background

Wavelets in Geology and Geophysics

- Fast discrete curvelet transform, 2006. [8]



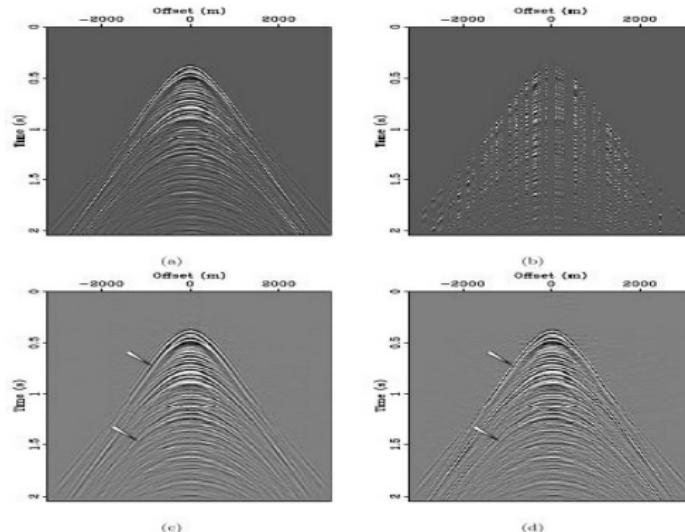
a) Delta function b) Approximate solution at $t = 0.25\text{s}$ c) Approximate solution at $t = 0.75\text{s}$

Image size	$T_{Fwd}(\text{s})$	$T_{Adj}(\text{s})$	$T_{Inv}(\text{s})$	T_{Fwd}/T_{FFT}	ℓ^2 error
128×128	0.088832	0.091578	1.006522	24.6756	1.4430e-06
256×256	0.376838	0.390533	4.002353	19.0322	8.8154e-07
512×512	2.487052	2.579102	35.09599	18.2202	5.3195e-07
1024×1024	16.47702	16.87764	129.3631	28.9579	3.2390e-07
2048×2048	62.42980	65.09365	566.1732	24.1920	3.4305e-06

Background

Wavelets in Geology and Geophysics

- Fast discrete curvelet transform - reconstruct seismic data, 2007.
[42]

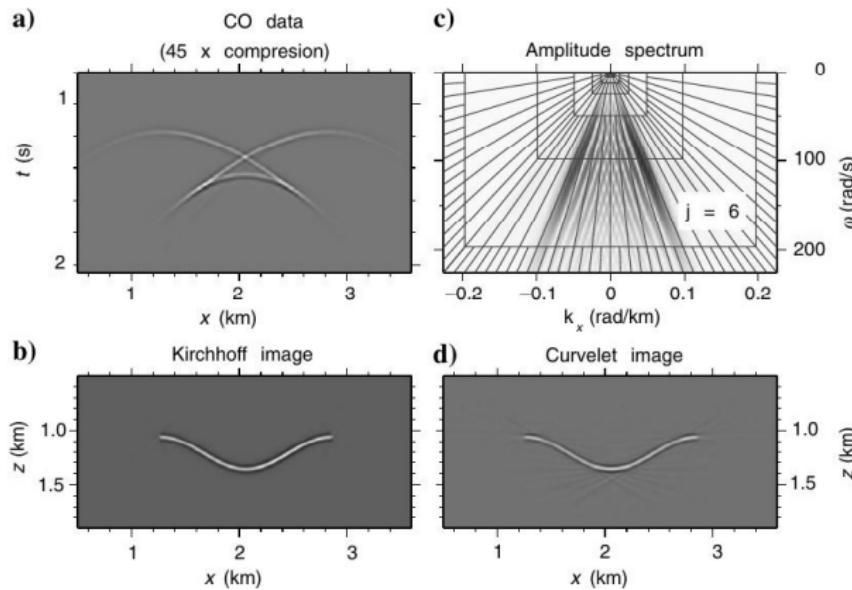


a) Fully sampled real data shot gather b) Randomly subsampled shot gather with 80% of the traces missing in the receiver and shot directions c) Curvelet-based recovery d) Curvelet-based recovery with focusing

Background

Wavelets in Geology and Geophysics

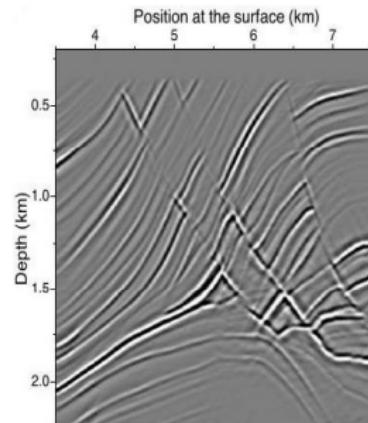
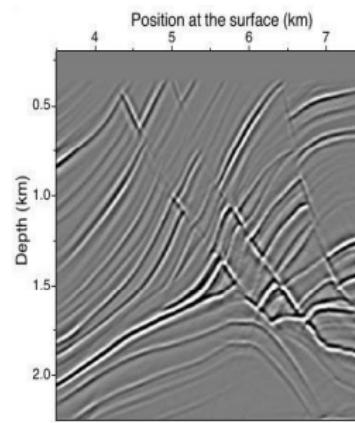
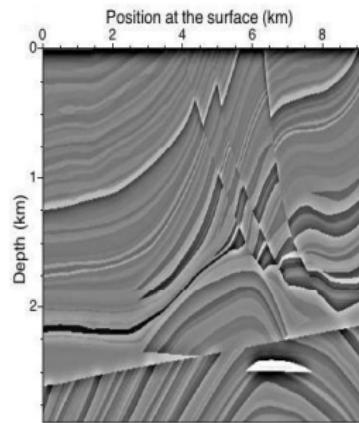
- Zero offset Kirchhoff migration with curvelets, 2007. [20]



Background

Wavelets in Geology and Geophysics

- Seismic demigration/migration with curvelets, 2008. [10]



Current works

- **Study of seismic migration methods.**
- Generation and Migration of 2D synthetic seismic data using seismic unix.
- Forward modeling of 2D acoustic wave equation using finite differences method (second order in time and second order in space).
- Forward modeling of 2D acoustic wave equation using the pseudospectral method (second order in time, second, fourth and sixth order in space).

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- C language implementation for Phase shift migration.
- The Haar system.
- Discrete approximation of a function using the Haar system.

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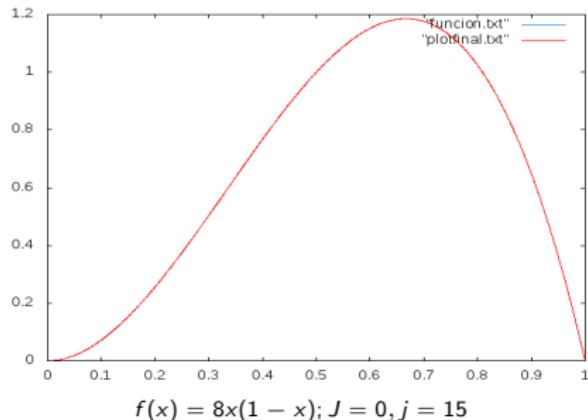
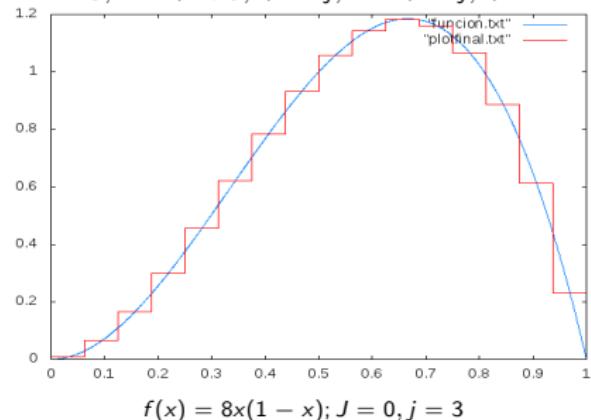
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Current works

Haar system

$$f(x) = \sum_{k=0}^{2^J-1} a_{J,k} p_{J,k}(x) + \sum_{j=J}^{\infty} \sum_{k=0}^{2^j-1} b_{j,k} h_{j,k}(x) \quad (1)$$

where $a_{J,k} = \langle f, p_{J,k} \rangle$, $b_{j,k} = \langle f, h_{j,k} \rangle$



Current works

- C language implementation for Phase shift migration.
- The Haar system.
- Discrete approximation of a function using the Haar system.
- Discrete Haar Transform.

Discrete Haar transform

Approximation and detail matrix

Given $L \in N$ even, define the $(L/2) \times L$ matrices H_L and G_L by

$$H_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & 1 & 0 & \dots & 0 \\ & & & & \ddots & & \\ 0 & & & \dots & 0 & 1 & 1 \end{pmatrix} \quad (2)$$

Discrete Haar transform

Approximation and detail matrix

$$G_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ & & & & \ddots & & \\ 0 & & & & \dots & 0 & 1 & -1 \end{pmatrix} \quad (3)$$

The matrix H_L is referred as the approximation matrix, the matrix G_L as the detail matrix.

Wavelet matrix

Define $L \times L$ matrix W_L by

$$W_L = \begin{pmatrix} H_L \\ G_L \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 1 & 0 & \dots & 0 \\ & & & & \ddots & & \\ 0 & & & \dots & 0 & 1 & 1 \\ 1 & -1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ & & & & \ddots & & \\ 0 & & & \dots & 0 & 1 & -1 \end{pmatrix} \quad (4)$$

1D Discrete Haar transform

If we consider a initial sequence of data, a_0 , to be a vector of length $L = 2^N$, $N \in \mathbb{N}$

$$a_0 = (a_0(0), a_0(1), \dots, a_0(2^N - 1))$$

The discrete Haar transform (DHT) of a_0 is given by

$$\begin{pmatrix} c_j \\ d_j \end{pmatrix} = W_L a_0 = \begin{pmatrix} H_L \\ G_L \end{pmatrix} a_0 = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{\frac{L}{2}-1} \\ d_0 \\ d_1 \\ \vdots \\ d_{\frac{L}{2}-1} \end{pmatrix}$$

1D Discrete Haar transform

c_j is called the averages block.

$$c_j = H_L a_0 = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{\frac{L}{2}-1} \end{pmatrix}$$

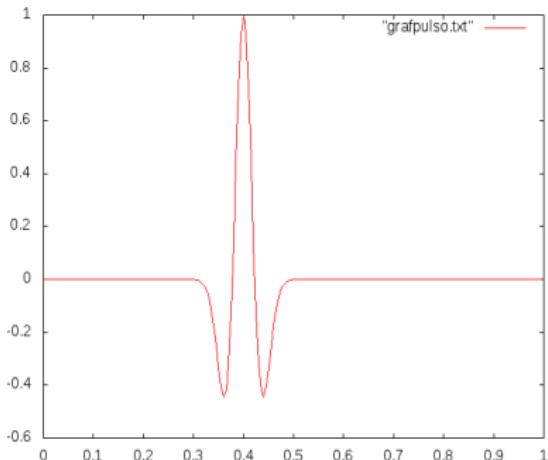
d_j is called the details block.

$$d_j = G_L a_0 = \begin{pmatrix} d_0 \\ d_1 \\ \vdots \\ d_{\frac{L}{2}-1} \end{pmatrix}$$

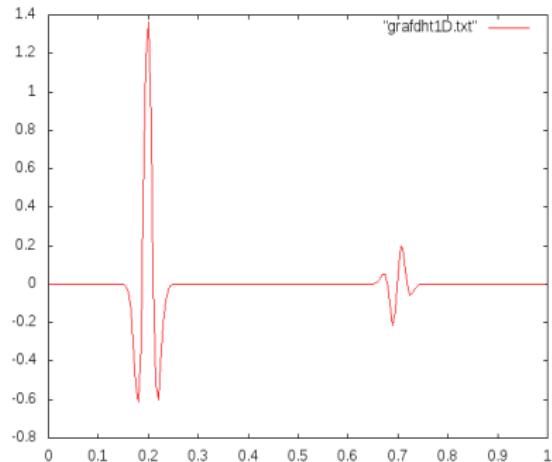
DHT Ricker Pulse

$$f(t) = (1 - 2\pi fc^2(t - 0.4)^2)e^{-\pi^2 fc^2(t - 0.4)^2} \quad (5)$$

with $fc = 5 \text{ hz}$ y $0 \leq t < 1$



Original Ricker pulse



DHT Ricker pulse

2D Discrete Haar transform

2D Discrete Haar transform

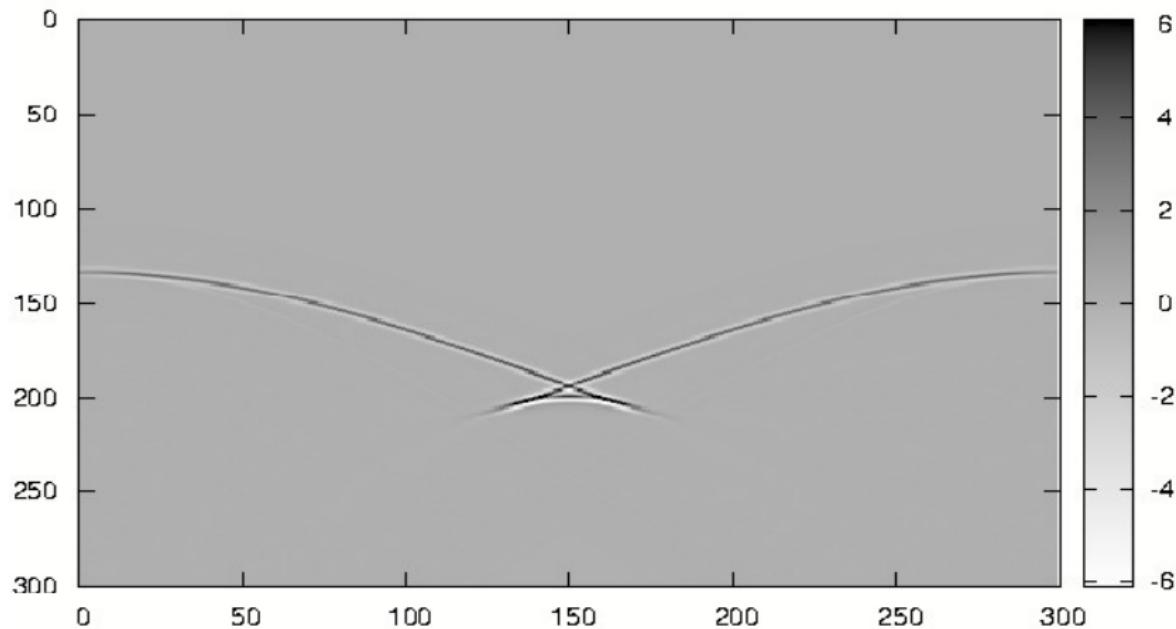
Suppose that A is an $M \times N$ matrix where M, N are even positive integers. The two-dimensional discrete Haar wavelet transformation of $M \times N$ matrix A is defined as

$$B = W_M A W_N' \quad (6)$$

where W_M, W_N are defined by (4)

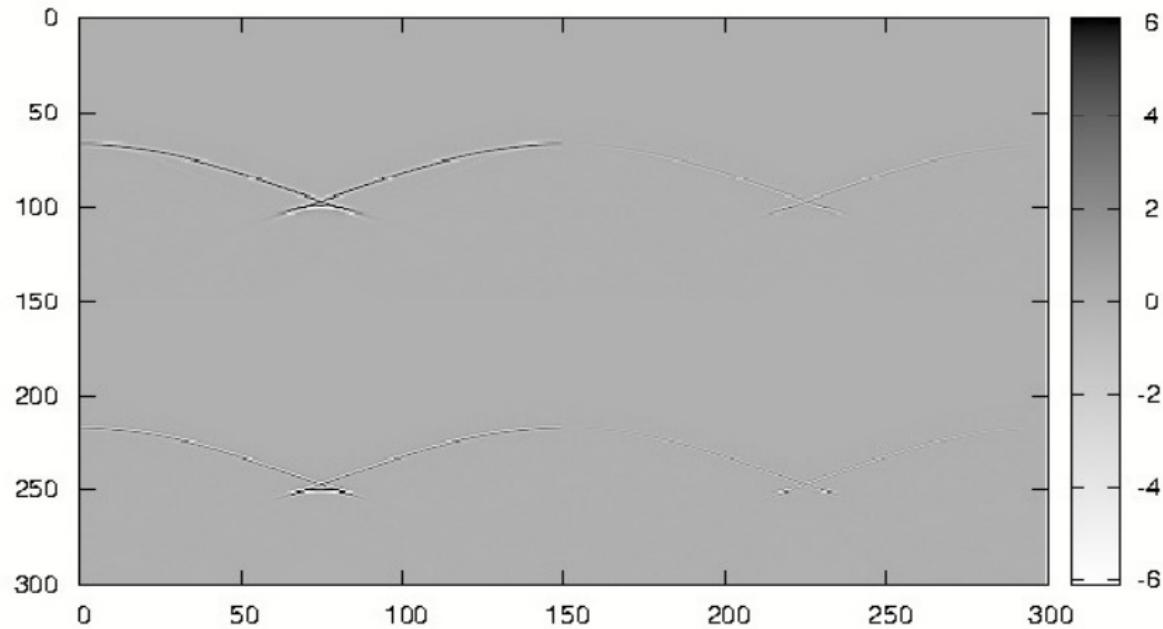
Sinclina model

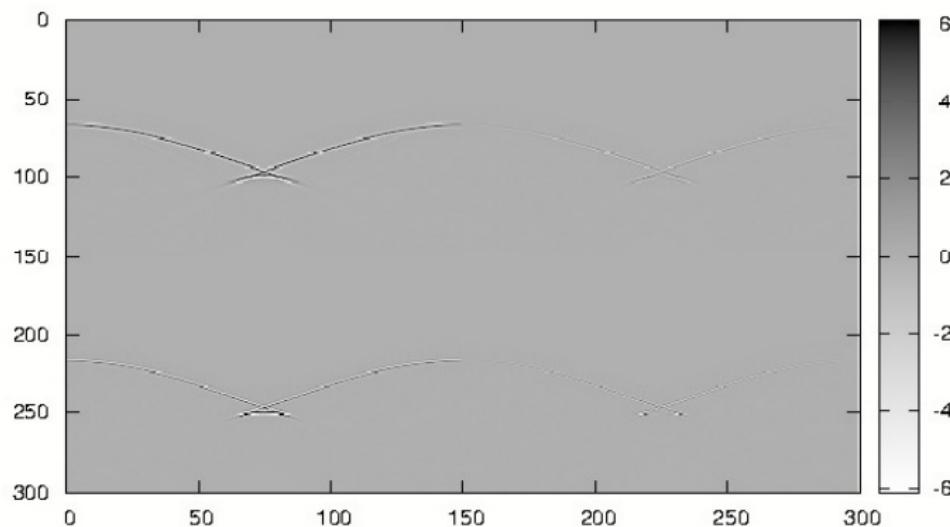
Zero offset seismic section



DHT Sinclina model

DHT Zero offset seismic section





$$H_L A H_L'$$

$$H_L A G_L'$$

$$G_L A H_L'$$

$$G_L A G_L'$$

Work perspectives

Is it possible to calculate the Laplacian (2D, 3D) using a different method that does not use a finite differences scheme or pseudospectral?

Work perspectives

Is it possible to improve the dispersion in wave propagation using a different method to calculate the Laplacian (2D, 3D), for example, a wavelet transform or another transform?

Work perspectives

Is it feasible to find other 2D, 3D orthogonal transformation that can be used in another method of migration, such as reverse time migration, allowing improve the images obtained through the analysis of the signals recorded on the surface?

Work perspectives

Search for a methodology to find the subsurface velocity field (2D, 3D) through the analysis of the signals recorded on the surface.

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